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A Fan-tastic Quantitative Exploration of Ohm's Law

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Teaching simple circuits and Ohm's law to students in the introductory classroom has been extensively investigated[1-4] through the common practice of using incandescent light bulbs to help students develop a conceptual foundation before moving on to quantitative analysis.[5,6] However, the bulb filaments' resistance has a large temperature dependence,[6-8] which makes them less suitable as a tool for quantitative analysis. Some instructors show that light bulbs do not obey Ohm's law either outright or through inquiry-based laboratory experiments.[9] Others avoid the subject altogether by using bulbs strictly for qualitative purposes and then later switching to resistors for a numerical analysis, or by changing the operating conditions of the bulb so that it is "barely" glowing.[6] It seems incongruous to develop a conceptual basis for the behavior of simple circuits using bulbs only to later reveal that they do not follow Ohm's law. Recently, small computer fans were proposed as a suitable replacement of bulbs for qualitative analysis of simple circuits where the current is related to the rotational speed of the fans.[10] In this contribution, we demonstrate that fans can also be used for quantitative measurements and provide suggestions for successful classroom implementation.

Effective resistance of a fan

Ohm's law, $I = \Delta V/R$, states that the current (I) through a circuit element is directly proportional to the potential difference (ΔV) applied to it, and inversely proportional to the resistance (R). Rearranging this equation gives the resistance as $R = \Delta V/I$. However, in order to characterize a circuit element with a well-defined resistance, the ratio between the potential difference and current must remain unchanged for different values of potential difference. Elements for which this is true are considered "ohmic" and can be investigated using Ohm's law. If the current through an ohmic device is plotted against the applied voltage, the relationship should be linear and the inverse of the slope would be the resistance. With this in mind, the inverse of the resistance should be defined as the ratio of the change in the potential difference to the change in current. If such a linear relationship exists between the applied voltage and the current through a fan, it can be considered ohmic and can be characterized by an "effective" resistance.

A particular fan model (ADDA - AD0405MB-C50) was previously identified to be favorable for qualitative experiments.[10] As such, this fan model was chosen for the quantitative experiments presented in this work.[11] Once we attached these fans to a standard power supply, the applied voltage and current can be measured using digital multimeters (DMMs); specifically, a Fluke 87 III for current and a Fluke 114 for voltage were used in this work. A limitation of using fans is that it is not possible to measure their effective resistance when they are not in operation.

When considering how to plot the data, it is important to consider the pedagogical implications. The applied voltage could be plotted against current, which would be represented by rearranging Ohm's law as $\Delta V = IR$, another common form. However, several authors have suggested that this form of Ohm's law can lead students to incorrectly associate the current as the cause of potential difference across a resistive element, rather than being a consequence of the potential difference.[12-14] In addition, since the applied voltage was the independent variable and the current was measured, it is not proper to put the voltage on the y-axis. For these reasons, $I = \Delta V/R$ has been suggested as the better variation for student understanding,[14] and our data were plotted with the current against the applied voltage in Fig. 1. As shown in the figure, the response of the fan within its operational voltages, from 2.5 V to 7.5 V, is quite linear. This implies a constant effective resistance, which means that the fans can be considered ohmic. As mentioned above, the slope of a linear fit of this data would give a value for the inverse of the resistance of the fan, which was found to be 38.3 Ω for the data plotted in Fig. 1. Since the fans exhibit ohmic behavior, they are treated as simple resistive circuit elements, ignoring their actual wiring and internal workings for the remainder of this article.

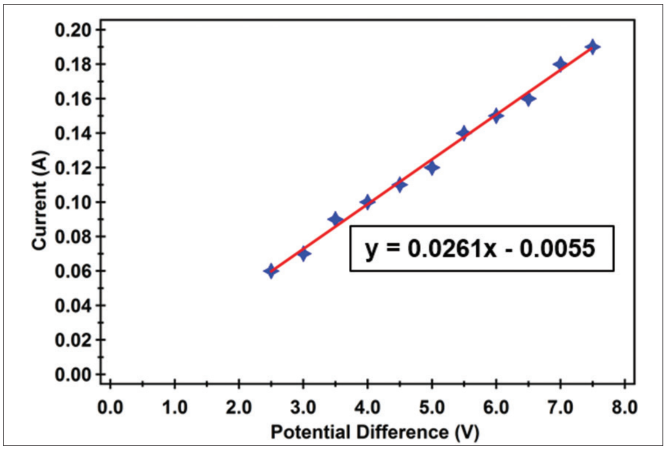

Fig. 1. Plot of current vs. voltage for an ADDA model fan. After a linear fit of the data, the fan's effective resistance can be taken as the reciprocal of the slope. From this data set, the effective resistance of the fan was found to be 38.3 Ω.

To observe the "ohmic" behavior, it is important to be above the threshold voltage, where it starts to spin. These

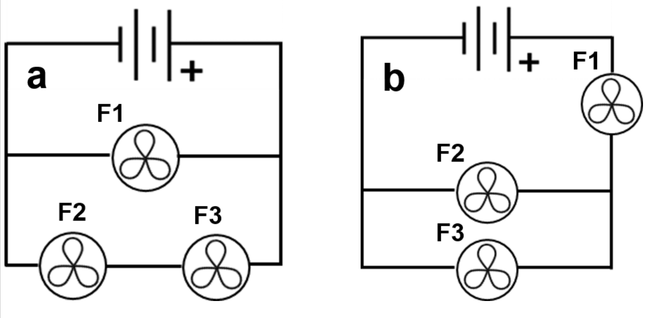

Fig. 2. Circuit diagrams for the combination circuits with (a) two fans in series with one in parallel and (b) two fans in parallel with one in series.

measurements were repeated for eight fans purchased at different times. From these measurements the effective resistance of the fans was determined to be 37.9 Ω ± 3.2%. The effective resistance was consistent within 5% across all fans, which is comparable to the tolerance of a standard carbon resistor. In the data sheet from the manufacturer (ADDA),[15] the power for the fan is listed as 0.8 W for a current of 0.16 A, which is equivalent to a resistance of 31 Ω using $P = I^2R$. There is only one significant figure on the power, which indicates that our measurement is reasonable.

A few lessons that helped minimize error were learned while taking data. As with any experiment, the precision and internal resistance of the DMM can influence the results. The orientation of the fan (upward, downward, or sideways) can make a difference on the measured current for the same applied voltage. For example, one fan with 7.5 V applied will measure a current of 0.198 A sideways, 0.206 A upward, and 0.176 A downward. It is assumed that the internal resistance changes when the flow of the air through the fan is interrupted or when a force is put on the fan blades. All measurements reported were taken when the fans were sideways. The threshold voltage for the fans to begin spinning is ~1.5 V. When operating slightly above the threshold voltage of ~1.5 V, the fans do not exhibit a steady or measurable current although they are spinning. When the fan exceeds its maximum voltage of ~8.0 V, the fans will typically smoke and smell. Afterward, some fans may be permanently damaged and fail to rotate, though others may continue to function. Thus, if 8.0 V or more is available, this condition should be avoided so that spurious data are not collected. As the fan resistances are on the order of 10 Ω, the choice of wire connectors also needed to be considered as some of the less expensive, braided, and higher overall gauge connecting wires can measure as much as 1 to 2 Ω in resistance independently. Finally, it is important to note that the fans used in this work, along with most PC cooling fans, are brushless and electronically commutated. Thus, the current drawn fluctuates over time and the DMM measures an average current. When less precise DMMs are used to make these measurements, there is not a significant impact on the results.

Table I. Summary of the results for the average resistance of parallel and series circuits using the ADDA fans with a supply voltage of 6.0 V.

Parallel			Series		
# Fans	Total Current	Average Resistance	# Fans	Total Current	Average Resistance
1	0.155 A	38.7 Ω	1	0.155 A	38.7 Ω
2	0.315 A	38.1 Ω	2	0.074 A	40.5 Ω
3	0.486 A	37.0 Ω	3	0.057 A	35.1 Ω

Series, parallel, and combination circuits

After demonstrating the ohmic behavior of the fans within their operational range, the next step is to explore the consistency of this behavior in circuits with more than one fan. First, quantitative measurements were collected by wiring the fans in both series and parallel circuits, with two and three fans. The results were compared to those for a single fan under the same applied voltage. The effective resistances that were calculated in these experiments were also compared to those found in the previous section. For series circuits, it was important to choose an overall supply voltage that led to potential differences across each fan that were above the threshold voltage. If the potential drop across a single fan was not above its threshold voltage, then it would not self-start, and the measured current values were not stable and would vary significantly. For parallel circuits, on the other hand, it was important to choose voltages that would not burn out the fans. In both types of circuits, a supply voltage in the range of 6.0 to 7.5 V worked well for one to three fans. In Table I, results for a 6.0-V voltage setting are presented, as this voltage is the same as the combined voltage of either four D-cell batteries in series or a 6-V lantern battery. For each supply voltage, the potential difference across each fan and the currents through them were measured using the same DMMs as in the previous section. The current and voltage values did not fluctuate within the mA and mV scale during the measurement, respectively, with an exception of three fans in series, where the current varied by ±5 mA. Since the fans were shown to be ohmic, $R = \Delta V/I$ can be used to calculate the resistance at individual values of potential difference. Using this equation and the properties of resistors that are connected in series and in parallel, the average single fan resistance was calculated for each circuit. For example, if we assume that each fan has the same average resistance, then the circuit with three fans connected in parallel would have a total resistance of $R_{avg}/3$. If you multiply the total resistance that was calculated by three, you get a value for the average resistance of one fan.

Next, two common combination circuits were wired using the fans. Figure 2 shows the configurations of (a) two fans in series with one in parallel and (b) two fans in parallel with one in series. The resistances for each fan were determined by applying Ohm's law. These values were then compared to the individual resistances of the fans that were determined experimentally when they were connected only in series or only

Table II. Summary of the results for a quantitative analysis of two combination circuits using the ADDA fans with a supply voltage of 6.0 V.

Two Series/One Parallel				Two Parallel/One Series			
Fan	Voltage Drop	Current	Resistance Calculated	Fan	Voltage	Current	Resistance Calculated
F1 (P)	5.96 V	0.155 A	38.5 Ω	F1 (S)	4.3 V	0.116 A	37.1 Ω
F2 (S)	2.88 V	0.079 A	36.5 Ω	F2 (P)	1.7 V	0.054 A	31.5 Ω
F3 (S)	3.07 V	0.0079 A	38.9 Ω	F3 (P)	1.7 V	0.054 A	31.5 Ω

Table III. Summary of the results for a quantitative analysis of two combination circuits using the ADDA fans with a supply voltage of 7.5 V.

Two Series/One Parallel				Two Parallel/One Series			
Fan	Voltage Drop	Current	Resistance Calculated	Fan	Voltage	Current	Resistance Calculated
F1 (P)	7.45 V	0.196 A	38.0 Ω	F1 (S)	5.05 V	0.135 A	37.4 Ω
F2 (S)	3.81 V	0.097 A	39.3 Ω	F2 (P)	2.55 V	0.067 A	38.1 Ω
F3 (S)	3.65 V	0.097 A	37.6 Ω	F3 (P)	2.55 V	0.068 A	37.4 Ω

Table IV. Summary of the measurements for a quantitative analysis of two fans with different effective resistances when connected in series and in parallel.

Parallel				Series			
Voltage Total	Current Total	Current HRF	Current LRF	Current Total	Voltage Both	Voltage HRF	Voltage LRF
7.5 V	0.46 A	0.19 A	0.26 A	0.12 A	7.34 V	4.4 V	2.93 V
6.0 V	0.37 A	0.16 A	0.22 A	0.09 A	5.89 V	3.32 V	2.56 V

Table V. Summary of the resistance calculations for a quantitative analysis of two fans with different effective resistances when connected in series and in parallel.

Resistances – Parallel					Resistances – Series				
Voltage	HRF	LRF	Total (exp)	Total (calc)	Voltage	HRF	LRF	Total (exp)	Total (calc)
7.5 V	39.9 Ω	28.8 Ω	16.3 Ω	16.7 Ω	7.5 V	36.7 Ω	24.4 Ω	61.1 Ω	61.1 Ω
6.0 V	37.5 Ω	27.3 Ω	16.2 Ω	15.8 Ω	6 V	36.9 Ω	28.4 Ω	65.3 Ω	65.3 Ω

in parallel. The results for a supply voltage of 6.0 V and 7.5 V are summarized in Tables II and III. These results agree with the qualitative observations that were previously reported for the same combination circuits in Ref. 10. For example, with a combination circuit where two fans are in parallel with one in series, you would expect the speed of the fans in parallel to be the same and significantly slower than the fan in series, which was observed.[10]

Overall, the agreement between the resistances that were measured in series, parallel, and combination circuits differ by at most 10%. The biggest error involved the average measurement for three fans in series. As mentioned earlier, the current draw from these fans is not purely DC, and, for this arrangement in particular, a significant current fluctuation (± 5mA) was observed. It should also be noted that if the measurements are made just above the threshold voltage, these errors can jump to nearly 20%, which, however, does not have a significant effect on the qualitative observations.[10] This behavior was observed for the two in parallel with one in series data for a 6.0-V supply voltage, where the voltage across the two fans in parallel is 1.7 V and the calculated resistance is only 31.5 Ω. When the supply voltage is increased to 7.5 V, however, the effective resistance of these two fans returned to the expected range. Overall, when within operational range, the experimental values are in good agreement with those calculated using Ohm's law, which also validate the qualitative observations previously reported.[10]

Fan circuits with different effective resistances

A different fan model (Multicorp - MC36256) was also identified to be capable of demonstrating simple circuit behaviors.[10] This fan model has a higher threshold voltage (~3 V) than the ADDA fans; however, once the supply voltage sufficiently exceeded this value, an ohmic behavior was observed. After plotting the VI curve and taking the slope, the Multicorp fans yielded a consistent effective resistance of 27.1 Ω ± 5%. Knowing that two types of fans possess different effective resistances opens up several options for combination circuits. This idea is analogous to the use of light bulbs with different resistances for qualitative comparisons, which are found in the CASTLE Project kits.[16,17] However, since fans exhibit an ohmic behavior, quantitative analysis can be performed for complex combination circuits while utilizing both of the different fan models in the same circuit.

For these experiments the 27.1 Ω Multicorp and 37.9 Ω ADDA fans will be referred to as low resistance fan (LRF) and high resistance fan (HRF), respectively. One LRF and one HRF were connected in a series circuit and also in a parallel circuit. Once again, the voltage across each fan and the currents through them were recorded with a DMM for supply voltages of 6.0 V and 7.5 V. The data collected with the DMM are summarized in Table IV. Using Ohm's law, the resistances of each fan were calculated and given in Table V. These results confirmed that the two fans maintained comparable effective resistances to those measured individually while wired in combination circuits.

As the frames for these fans are different sizes, the speed of the fan blades cannot be directly compared as they were in Ref. 12, and thus qualitative comparisons are not possible. It may be possible that alternative air flow models of the ADDA fan could allow for the qualitative comparisons; however, the purchase of these was cost prohibitive due to large purchase order requirements and long lead times. As fans are further adopted, we expect that other models will be found that exhibit different effective resistances and are more readily available. It would be quantitatively and qualitatively beneficial to have a fan that possesses twice the resistance in addition to fans that have a similar operating voltage range.

Conclusions

Computer fans have been successfully used to qualitatively explore Ohm's law and simple circuits by using the effective resistance of each fan. Ohm's law was verified for both series and parallel circuits containing two fan models, as well as series, parallel, and combination circuits that use only a single-fan model. Previous qualitative observations for a single fan were verified quantitatively as new qualitative observations for two fans were explored. These attributions also open new avenues for quantitative analysis. Photogates or high-speed video analysis, for example, could be used to measure the fan speed or, as another example, a sound analyzer could be used to measure the fan's frequency as a function of the fan blades' speed. These alternative analyses could provide further insight into basic circuits and experimental methods. Though the fan is a complex circuit element, these and previous results[10] demonstrate both the qualitative and quantitative benefits of using fans as an effective tool to teach simple circuits. Some limitations do exist such as a required threshold voltage; however, these can easily be overcome by identifying the proper operating range. As bulbs do not allow for these simplistic quantitative measurements, fans are arguably a suitable, if not better, replacement for light bulbs.

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