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Teaching Beyond the Assessment

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Abstract: This paper discusses how students can learn mathematics assessed on standardized testing in a way that promotes deeper thinking about the mathematics. While potential assessments aligned to the Common Core Content Standards drive the focus of the curriculum, technology can provide a way for delving into conceptual understanding. Using two standardized test questions, this paper provides examples of how technology can promote deeper thinking about mathematical concepts than what is supposed to be assessed in the questions.

Standardized assessments have long driven the content taught in mathematics classes. If a topic was not on the assessment, then it was not emphasized in the classroom. (Jennings & Stark Rentner, 2006) However, the Common Core State Standards (CCSS) in Mathematics (NGA, 2010) notes, "Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness" (p. 4). So, while a question on a standardized assessment may address a particular skill, the understanding behind that skill is also valuable for students to meet college and career readiness. Additionally, teachers cannot prepare students for every possible question on a standardized assessment. Therefore, understanding of a topic is critical for students to be able to apply their knowledge in solving novel problems, but also to be able to apply or use that knowledge beyond the classroom. "Students who lack understanding of a topic may rely on procedures too heavily. Without a flexible base from which to work, they may be less likely to consider analogous problems, represent problems coherently, justify conclusions, apply the mathematics to practical situations, use technology mindfully to work with the mathematics, explain the mathematics accurately to other students, step back for an overview, or deviate from a known procedure to find a shortcut." (NGA, 2010, p. 8)

The idea of learning mathematical habits of mind is present in the National Council of Teachers of Mathematics (NCTM) Process Standards (2000) and in the CCSS of Mathematical Practice (NGA, 2010). Incorporating the ideas of the Mathematical Practices is essential for today's students to be successful in college and the workplace. This means, even though teachers and students are held accountable by the results of standardized assessments, classroom teachers must attempt to teach beyond the assessments. One possible way to achieve this is to select standardized test questions that allow for the use of the technology to launch a deeper exploration of the concept in the problem.

Patterns in Area

The following multiple-choice problem is from a test preparation book for the New Jersey High School Proficiency Assessment (*HSPA Power! Mathematics*, 2007):

The area of a circle with diameter 10 inches is greater than the combined area of a circle with diameter 8 inches and a circle with diameter 2 inches by approximately how many square inches?

This problem would be considered a challenging problem because students must use the formula for the area of a circle three times. After finding the three areas, which requires halving the given diameters, students must determine which areas to sum and then find the difference between the largest area and the calculated sum of the two smaller areas.

However, as stated, this problem focuses on procedural skills of calculating area of a circle and determining whether to add or subtract the values. If we revise the question to explore the pattern between the differences when the diameters of the two smaller circles are changed, we can explore ideas beyond the "assessed concept". The new question would be:
Is there a pattern in the difference between the area of a circle with diameter 10 and two smaller circles with diameters that sum to 10?

Students can select different diameters that sum to ten, such as 1 and 9 or 3 and 7, and find the areas. They still practice the skill, but now have a different goal to achieve. Upon finding all pairs, students can develop the table in Figure 1, which shows the diameter of one smaller circle and the difference between the area of the larger circle and sum of the two smaller circles.

	A diam1	B areadiff	C	D	E	F
=	=capture('r1	=capture(ad				
1	0.	0.				
2	1.	14.1372				
3	2.	25.1327				
4	3.	32.9867				
5	4.	37.6991				
6	5.	39.2699				
7	6.	37.6991				
8	7.	32.9867				
9	8.	25.1327				
10	9.	14.1372				
11	10.	0.				

A1 =0.

Figure 1: Data

Using graphing technology, students can see the points appear to form the parabola shown in Figure 2.

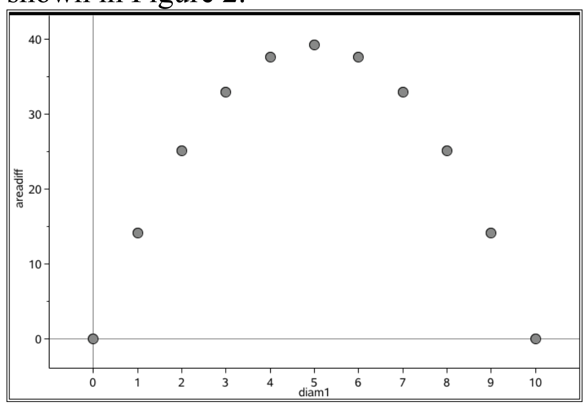

Figure 2: Graph of Data

Using the regression features of the calculator students learn the function for the data is $f(x) = 1.5708x^2 + 15.708x$. The next question is whether or not the coefficients have any meaning for students, who hopefully recognize the coefficient of

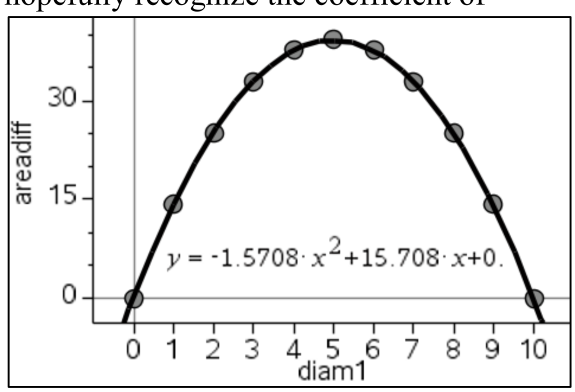

Figure 3: Regression Equation

the square term is π/2 and the coefficient of the linear term is 10(π/2), where 10 is the diameter of the largest circle. Pushing the relationship even further, it can be asked whether or not the relationship is true for any given diameter, d, of a circle and two smaller circles whose diameters sum to the given diameter. An algebraic exploration reveals the following:

$$Area = \pi\left(\frac{d}{2}\right)^2 - \left(\left(\pi\left(\frac{x}{2}\right)^2\right) + \left(\pi\left(\frac{d-x}{2}\right)^2\right)\right)$$

$$= \pi\left(\frac{d^2}{4}\right) - \left(\pi\left(\frac{x^2}{4}\right) + \pi\left(\frac{d^2}{4}\right) - 2\pi\left(\frac{d}{2}\right)\left(\frac{x}{2}\right) + \pi\left(\frac{x^2}{4}\right)\right)$$

$$= -2\pi\left(\frac{x^2}{4}\right) + 2\pi\left(\frac{d}{2}\right)\left(\frac{x}{2}\right)$$

$$= -\left(\frac{\pi}{2}\right)x^2 + \left(\frac{\pi}{2}d\right)x$$

The last line is in the form that generalizes the findings from our example with diameter ten.

As this example shows, it is possible to use a question meant for a standardized assessment and explore the conceptual understanding behind the problem. By looking for patterns in all possible versions of the question, students repeatedly practice the skills needed to solve the problem, but more importantly for the purpose of looking for a general relationship. The use of technology allows students to visually examine their results and explore the underlying relationship. This deeper exploration also includes several other mathematical concepts and shows students connections between statistics, geometry and algebra.

The Pentagon Task

In elementary and middle school, students learn formulas for areas of triangles, quadrilaterals, circles and irregular shapes composed of other familiar shapes. However, learning area from a formula approach treats it as a static value. The 8th grade 2005 National Assessment of Educational Progress (NAEP) asked students to find the area of a given pentagon on a grid and then draw another non-congruent pentagon with the same area.

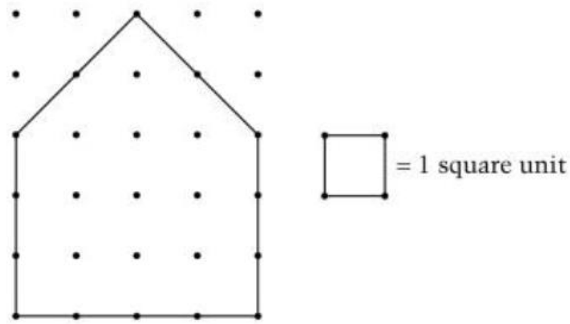

Figure 4: Original Pentagon

Questions:
a. What is the area, in square units, enclosed by the pentagon in the figure above?

b. On the figure below, draw a different pentagon that has the same area as the one shown. (Be sure the pentagon that you draw does not look the one shown when it is turned in a different direction.)

Figure 5: Grid to Draw New Pentagon

(NCES, 2005)

Only 2% of students correctly answered both parts of this question correctly and 48% received partial credit by finding the correct area of pentagon. (NCES, 2005)

The use of technology in exploring area could provide students with a more dynamic view of the subject. By recreating this problem on the TI-*n*spire handheld technology or other dynamic geometry software, students are able to explore different shapes that have the same area as the given pentagon. Two possible solutions are shown in Figures 6 and 7. These examples are selected to demonstrate the underlying mathematics of creating a figure with the same area.

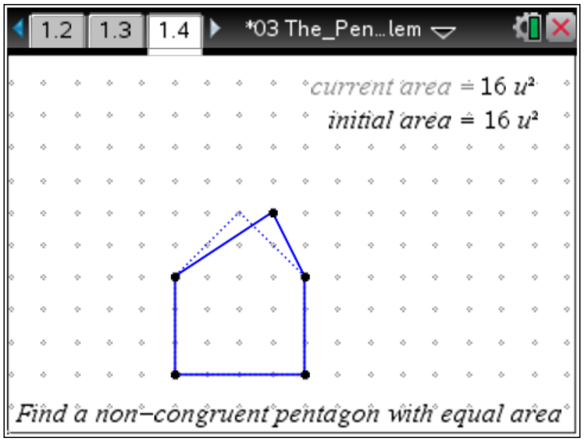

Figure 6: Pentagon Solution Example 1

In Figure 6, the student has moved the top vertex of the pentagon and created a non-congruent pentagon with the same area. One possible path of mathematical reasoning is the pentagon shown can be thought of as a rectangle with a triangle on top. This student has not changed the rectangle so the area is the same. By moving the top vertex one grid point to the right, the height and base of the triangle have not changed either. Since the height and base of the triangle have not changed, the area of the triangle will be equivalent. Therefore, the entire pentagon will have the same area as the original pentagon.

In Figure 7, the bottom two vertices have been shifted to the left one grid point in order to create a pentagon with the same area. One possible path of mathematical reasoning is to once again consider the pentagon as composed of a rectangle and a triangle. In this case, the triangle has not

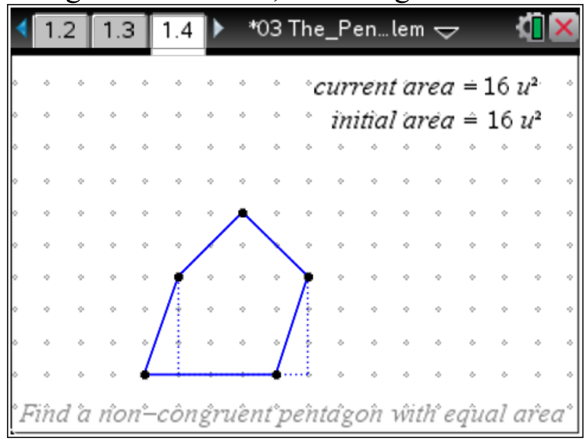

Figure 7: Pentagon Solution Example 2

changed so the area is the same. By shifting the bottom left vertex of the rectangle, a triangle has been added to the original rectangle, but the same triangle has been removed on the right side of the original rectangle when the bottom right vertex of the rectangle is shifted. Therefore, the area added on the left side is subtracted by the area on the right side producing a net area change of zero for the rectangle.

These two examples were selected to demonstrate two possible ways of creating a pentagon with equivalent area. First is to move vertices in such a way to keep the dimensions the same so the resulting area is unchanged. The second is to add and remove the same amount of area from the figure when the vertices are moved so the resulting area is the same. The example in Figure 8, shows the area added and removed do not need to be congruent figures as in the previous example. The area of the two

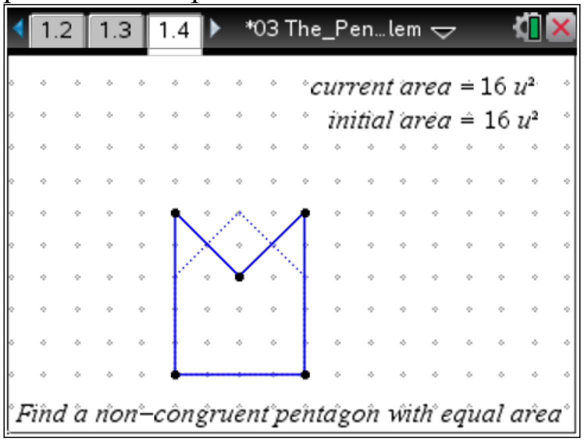

Figure 8: Pentagon Example 3

triangles formed on top of the rectangle have the same total area as the original triangle in order to create a pentagon with equivalent area. One possible justification is the height and base of the two new triangles is the same as the original triangle. Another justification could explain how a rectangle drawn around both the original triangle and the two new triangles would have the same area missing or filled.

The concepts of maintaining the dimensions of the original figure and conserving area can be very difficult to grasp if students do not explore area dynamically. By asking students the same question in a dynamic geometry environment, they have the opportunity to explore possibilities for finding an equivalent area and discuss their mathematical thinking and concepts in their solutions.

Pentagon Task and Pre-Service Teachers

When thirty pre-service middle school teachers (grade 4 – 8 certification) taking a pedagogy course were asked to solve the pentagon task on paper, only two were able to produce an alternate pentagon with the same area as the one shown. When given the dynamic version, every pre-service teacher developed a solution. While discussing the two strategies for creating a figure with equivalent area noted earlier, the class started to explore the question of how much the area changed as vertices of the pentagon moved. Two interesting lines of discussion developed. First, as the top vertex of the pentagon is moved up, the area of the pentagon (and triangle) changed by a total of two square units. After much discussion, the class concluded the area of the triangle changed at a rate equal to half the base (the base is fixed with length four) each time the height increased by one. Second, as the right vertices of the pentagon were both moved, the area of the pentagon changed by four square units. Three square units were contributed by the rectangle and one unit was contributed by the triangle. The rate of change in the area of the triangle was now half of the height (the height is fixed with length two). The concept of rate of change became the focus of the discussion as half of the fixed variable in the area formula A = .5*bh* became the slope. This became a powerful discovery for the pre-service teachers since most saw slope only as "rise over run" rather than as rate of change in various situations.

The vignette of pre-service teachers substantiates the need for a dynamic view of learning area formulas since college level students could not produce a solution to this problem mimicking the results of the eighth graders. Additionally, learning beyond the intended assessed concepts can also occur with teachers as they explore their understanding of mathematics at levels beyond the expectations of the assessment.

Conclusion

The application of technology to explore patterns in the mathematical content of the two example standardized assessment questions shows how it is possible to examine deeper mathematical relationship, but still address the skills needed for the "test". Exploring mathematics in a dynamic environment allows students to seek out patterns and relationships that are not obvious when the mathematics is asked in a static assessment question. While standardized assessments may continue to drive accountability measures for students and teachers, having students explore mathematics through the appropriate use of technology can develop mathematical habits of mind, which go far beyond getting the correct answer.

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